# EE 508 Lecture 23

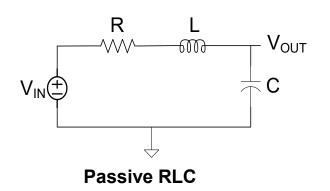
# Filter Synthesis Strategies

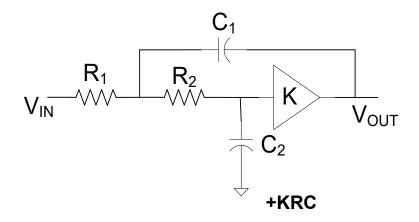
- Integrators

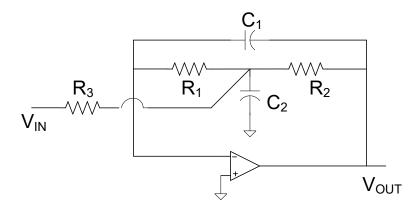
# **Sensitivity Comparisons**

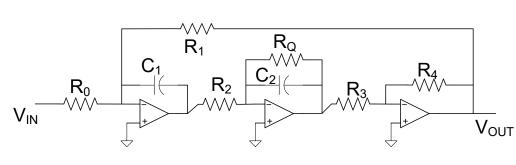
## Consider 5 second-order lowpass filters

(all can realize same T(s) within a gain factor)







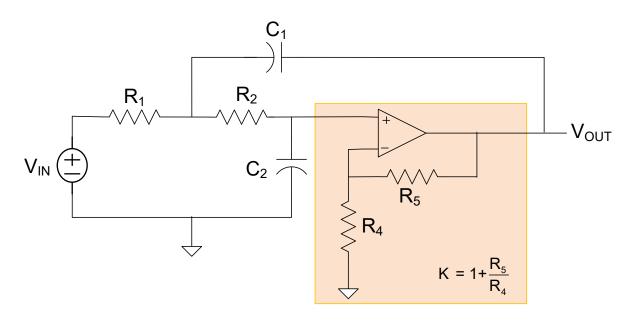


**Bridged-T Feedback** 

**Two-Integrator Loop** 

#### **Review from last time**

# b) + KRC (a Sallen and Key filter)



$$T(s) = \frac{\frac{K}{R_1 R_2 C_1 C_2}}{s^2 + s \left[ \left( \frac{1}{\sqrt{R_1 R_2 C_1 C_2}} \right) \left( \sqrt{\frac{R_1 C_1}{R_2 C_2}} + \sqrt{\frac{R_2 C_2}{R_1 C_1}} + \sqrt{\frac{R_1 C_2}{R_2 C_1}} - K \sqrt{\frac{R_1 C_1}{R_2 C_2}} \right) \right] + \frac{1}{R_1 R_2 C_1 C_2}$$

$$\omega_{0} = \sqrt{\frac{1}{R_{1}R_{2}C_{1}C_{2}}}$$

$$Q = \frac{1}{\left(\sqrt{\frac{R_{1}C_{1}}{R_{2}C_{2}}} + \sqrt{\frac{R_{1}C_{2}}{R_{1}C_{1}}} + \sqrt{\frac{R_{1}C_{2}}{R_{2}C_{1}}} - K\sqrt{\frac{R_{1}C_{1}}{R_{2}C_{2}}}\right)}$$

## How do these five circuits compare?

- a) From a passive sensitivity viewpoint?
  - If Q is small
  - If Q is large

- b) From an active sensitivity viewpoint?
  - If Q is small
  - If Q is large
  - If  $τω_0$  is large

## Comparison: Calculate all $\omega_0$ and Q sensitivities

#### Consider passive sensitivities first

a) - Passive RLC

$$S_R^{\omega_0} = 0$$

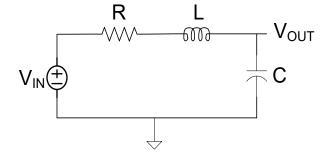
$$\mathbf{S}_L^{\omega_0} = -\frac{1}{2}$$

$$\mathbf{S}_C^{\omega_0} = -\frac{1}{2}$$

$$S_R^Q = -1$$

$$\mathbf{S}_C^Q = -\frac{1}{2}$$

$$\mathbf{S}_L^Q = \frac{1}{2}$$



$$Q = \frac{1}{R} \sqrt{\frac{L}{C}}$$

$$\omega_0 = \sqrt{\frac{1}{LC}}$$

## Case b1: +KRC Equal R, Equal C

$$\omega_0 = \sqrt{\frac{1}{R_1 R_2 C_1 C_2}}$$

$$Q = \frac{1}{\left(\sqrt{\frac{R_{1}C_{1}}{R_{2}C_{2}}} + \sqrt{\frac{R_{2}C_{2}}{R_{1}C_{1}}} + \sqrt{\frac{R_{1}C_{2}}{R_{2}C_{1}}} - K\sqrt{\frac{R_{1}C_{1}}{R_{2}C_{2}}}\right)}$$

$$S_{R_1}^{\omega_0} = S_{R_2}^{\omega_0} = S_{C_1}^{\omega_0} = S_{C_2}^{\omega_0} = -\frac{1}{2}$$

$$\mathsf{S}_{K}^{\omega_{0}}=0$$

$$\mathsf{S}_{R_1}^{\mathcal{Q}} = \mathcal{Q} - \frac{1}{2}$$

$$\mathsf{S}_{R_2}^{\mathcal{Q}} = -Q + \frac{1}{2}$$

$$S_{C_1}^Q = 2Q - \frac{1}{2}$$

$$S_{C_2}^Q = -2Q + \frac{1}{2}$$

$$S_K^Q = 3Q - 1$$

$$Q = \frac{1}{3 - K}$$

$$\omega_0 = \frac{1}{RC}$$

If Q<sub>N</sub>=10 in +KRC filter, what happens to Q if R<sub>1</sub> increases by 1%? By 10%?

$$\mathsf{S}_{R_1}^{\mathcal{Q}} = \mathcal{Q} - \frac{1}{2}$$

$$\frac{\Delta R_1}{R_1} = 0.01$$

$$\frac{\Delta Q}{Q} \simeq S_{R_1}^Q \bullet \frac{\Delta R_1}{R_1} \simeq \left(Q - \frac{1}{2}\right) (0.01) = 9.5 \bullet 0.01 = 0.095$$

$$Q_{PREDICT} = 10*(1+0.095) = 10.95$$

$$Q_{ACTUAL} = 11.04$$

$$\frac{\Delta R_1}{R_1} = 0.1$$

$$\frac{\Delta Q}{Q} \simeq S_{R_1}^Q \bullet \frac{\Delta R_1}{R_1} \simeq \left(Q - \frac{1}{2}\right) (0.01) = 9.5 \bullet 0.1 = 0.95$$

$$Q_{PREDICT} = 10*(1+0.95) = 19.5$$

Sensitivity analysis quite useful if  $\frac{\Delta x}{x}$  is small but not accurate when  $\frac{\Delta x}{x}$  is large

## Effects of R<sub>1</sub> variations on +KRC Filter

$$\mathsf{S}_{R_1}^{\mathcal{Q}} = \mathcal{Q} - \frac{1}{2}$$

	Q	10	
	K	2.9	
Pct Chang	Sens /Var	Predicted	Actual
	Pred	Q	Q
1	0.095	10.95	11.04382
2	0.19	11.9	12.31647
3	0.285	12.85	13.90259
4	0.38	13.8	15.93444
5	0.475	14.75	18.63082
6	0.57	15.7	22.3818
7	0.665	16.65	27.95697
8	0.76	17.6	37.11537
9	0.855	18.55	54.94898
10	0.95	19.5	104.8809
11	1.045	20.45	1053.565
12	1.14	21.4	-132.288
13	1.235	22.35	-62.5303
14	1.33	23.3	-41.0657
15	1.425	24.25	-30.6394
16	1.52	25.2	-24.478
17	1.615	26.15	-20.4088
18	1.71	27.1	-17.5206
19	1.805	28.05	-15.3644
20	1.9	29	-13.6931

	Q	2	
	K	2.5	
Pct Chang	Sens /Var	Predicted	Actual
	Pred	Q	Q
1	0.015	2.03	2.030278
2	0.03	2.06	2.061123
3	0.045	2.09	2.092555
4	0.06	2.12	2.124591
5	0.075	2.15	2.157253
6	0.09	2.18	2.19056
7	0.105	2.21	2.224533
8	0.12	2.24	2.259197
9	0.135	2.27	2.294573
10	0.15	2.3	2.330686
11	0.165	2.33	2.367563
12	0.18	2.36	2.405228
13	0.195	2.39	2.443712
14	0.21	2.42	2.483041
15	0.225	2.45	2.523248
16	0.24	2.48	2.564364
17	0.255	2.51	2.606423
18	0.27	2.54	2.649459
19	0.285	2.57	2.693509
20	0.3	2.6	2.738613

Q	25	
K	2.96	
Sens /Var	Predicted	Actual
Pred	Q	Q
0.245	31.125	33.0588
0.49	37.25	48.55531
0.735	43.375	90.6151
0.98	49.5	637.3774
1.225	55.625	-128.087
	Sens /Var Pred 0.245 0.49 0.735 0.98	Sens /Var Predicted Pred Q 0.245 31.125 0.49 37.25 0.735 43.375 0.98 49.5

## Case b2: +KRC Equal R, K=1

$$\omega_0 = \sqrt{\frac{1}{R_1 R_2 C_1 C_2}}$$

$$Q = \frac{1}{\left(\sqrt{\frac{R_{1}C_{1}}{R_{2}C_{2}}} + \sqrt{\frac{R_{2}C_{2}}{R_{1}C_{1}}} + \sqrt{\frac{R_{1}C_{2}}{R_{2}C_{1}}} - K\sqrt{\frac{R_{1}C_{1}}{R_{2}C_{2}}}\right)}$$

$$\mathbf{S}_{R_1}^{\omega_0} = \mathbf{S}_{R_2}^{\omega_0} = \mathbf{S}_{C_1}^{\omega_0} = \mathbf{S}_{C_2}^{\omega_0} = -\frac{1}{2}$$

$$\mathsf{S}_{\scriptscriptstyle K}^{\scriptscriptstyle \omega_0}=0$$

$$\mathsf{S}_{R_1}^\mathcal{Q}=0$$

$$S_{R_2}^Q = 0$$

$$\mathsf{S}_{C_1}^{\mathcal{Q}} = \frac{1}{2}$$

$$\omega_0 = \frac{1}{RC}$$

$$\mathbf{S}_{C_2}^{\mathcal{Q}} = -\frac{1}{2}$$

$$Q = \frac{1}{2} \sqrt{\frac{C_1}{C_2}}$$

$$\mathsf{S}_K^Q = 2Q^2$$

# c) Bridged T Feedback

$$\omega_0 = \sqrt{\frac{1}{R_1 R_2 C_1 C_2}}$$

$$\omega_{0} = \sqrt{\frac{1}{R_{1}R_{2}C_{1}C_{2}}} \qquad Q = \frac{1}{\left(\sqrt{\frac{C_{2}}{C_{1}}}\right)\left(\sqrt{\frac{R_{1}}{R_{3}}} + \sqrt{\frac{R_{2}}{R_{1}}} + \frac{\sqrt{R_{1}R_{2}}}{R_{3}}\right)}$$

For  $R_1=R_2=R_3=R$ 

$$S_{R_1}^{\omega_0} = S_{R_2}^{\omega_0} = S_{C_1}^{\omega_0} = S_{C_2}^{\omega_0} = -\frac{1}{2}$$
  $S_{R_3}^{\omega_0} = 0$ 

$$S_{R_3}^{\omega_0} = 0$$

$$\mathbf{S}_{R_1}^{\mathcal{Q}} = -\frac{1}{6}$$

$$S_{R_2}^Q = -\frac{1}{6}$$

$$\omega_0 = \frac{3Q}{RC_1}$$

$$\mathsf{S}_{R_3}^{\mathcal{Q}} = \frac{1}{3}$$

$$Q = \frac{1}{3} \sqrt{\frac{C_1}{C_2}}$$

$$\mathsf{S}_{C_1}^{\mathcal{Q}} = -\frac{1}{2}$$

$$\mathsf{S}_{C_2}^\mathcal{Q} = \frac{1}{2}$$

# d) 2 integrator loop

$$\omega_0 = \sqrt{\frac{R_4}{R_3} \cdot \frac{1}{R_0 R_2 C_1 C_2}} \qquad Q = \frac{R_Q}{\sqrt{R_0 R_2}} \sqrt{\frac{C_2}{C_1}}$$
For:  $R_0 = R_1 = R_2 = R$   $C_1 = C_2 = C$   $R_3 = R_4$ 

 $\omega_0 = \frac{1}{RC}$ 

 $Q = \frac{R_Q}{R}$ 

For: 
$$R_0 = R_1 = R_2 = R$$

$$S_{R_{1}}^{\omega_{0}} = S_{R_{2}}^{\omega_{0}} = S_{R_{3}}^{\omega_{0}} = S_{C_{1}}^{\omega_{0}} = S_{C_{2}}^{\omega_{0}} = -\frac{1}{2}$$

$$S_{R_{4}}^{\omega_{0}} = S_{R_{2}}^{\omega_{0}} = S_{R_{3}}^{\omega_{0}} = S_{C_{1}}^{\omega_{0}} = -\frac{1}{2}$$

$$S_{R_{1}}^{\omega_{0}} = S_{R_{2}}^{\omega_{0}} = S_{R_{3}}^{\omega_{0}} = S_{C_{1}}^{\omega_{0}} = -\frac{1}{2}$$

$$S_{R_{4}}^{\omega_{0}} = S_{C_{2}}^{\omega_{0}} = \frac{1}{2}$$

$$\mathsf{S}_{R_0}^{\mathcal{Q}}=0$$

 $S_{R_{0}}^{Q} = 1$ 

-KRC passive sensitivities d)

$$\omega_0 = \sqrt{\frac{1 + (R_1/R_3)(1+K) + (R_1/R_4)(1+R_2/R_3+R_2/R_1)}{R_1R_2C_1C_2}}$$

$$Q = \frac{\sqrt{\frac{1 + \left(R_{1}/R_{3}\right)\left(1 + K\right) + \left(R_{1}/R_{4}\right)\left(1 + R_{2}/R_{3} + R_{2}/R_{1}\right)}{R_{1}R_{2}C_{1}C_{2}}}{\left(1 + \frac{R_{1}}{R_{3}}\right)\left(\frac{1}{R_{1}C_{1}}\right) + \left(1 + \frac{C_{2}}{C_{1}}\right)\left(\frac{1}{R_{2}C_{2}}\right) + \left(\frac{1}{R_{4}C_{2}}\right)}$$

For 
$$R_1 = R_2 = R_3 = R_4 = R$$
,  $C_1 = C_2 = C$ 

$$Q = \frac{\sqrt{5 + K_0}}{5} \qquad \omega_0 = \frac{\sqrt{5 + K}}{R C}$$

$$\omega_0 = \frac{\sqrt{5 + K}}{R C}$$

$$S_{R_1}^{\omega_0} = -\frac{1}{25Q^2}$$

$$S_{R_2}^{\omega_0} = -\frac{1}{2} + \frac{1}{25O^2}$$

$$\mathbf{S}_{R_1}^{\omega_0} = -\frac{1}{25Q^2}$$
  $\mathbf{S}_{R_2}^{\omega_0} = -\frac{1}{2} + \frac{1}{25Q^2}$   $\mathbf{S}_{R_3}^{\omega_0} = -\frac{1}{2} + \frac{3}{50Q^2}$ 

$$\mathbf{S}_{C_1}^{\omega_0} = \mathbf{S}_{C_2}^{\omega_0} = -\frac{1}{2} \quad \mathbf{S}_{R_4}^{\omega_0} = -\frac{3}{50Q^2} \qquad \qquad \mathbf{S}_{K}^{\omega_0} = \frac{1}{2} + \frac{1}{10Q^2}$$

$$S_{R_4}^{\omega_0} = -\frac{3}{50Q^2}$$

$$S_K^{\omega_0} = \frac{1}{2} + \frac{1}{10Q^2}$$

$$S_{R_1}^Q = \frac{1}{5} - \frac{1}{25O^2}$$

$$S_{R_1}^{Q} = \frac{1}{5} - \frac{1}{25O^2}$$
  $S_{R_2}^{Q} = -\frac{1}{10} + \frac{1}{25O^2}$   $S_{R_3}^{Q} = -\frac{3}{10} + \frac{3}{50O^2}$ 

$$S_{R_3}^Q = -\frac{3}{10} + \frac{3}{50Q^2}$$

$$S_{R_4}^Q = \frac{1}{5} - \frac{3}{50O^2}$$

$$S_{C_2}^Q = -\frac{1}{10}$$

$$\mathsf{S}_{C_1}^{\mathcal{Q}} = \frac{1}{10}$$

$$S_{R_4}^{Q} = \frac{1}{5} - \frac{3}{50Q^2}$$
  $S_{C_2}^{Q} = -\frac{1}{10}$   $S_{C_1}^{Q} = \frac{1}{10}$   $S_K^{Q} = \frac{1}{2} - \frac{1}{10Q^2}$ 

## Passive Sensitivity Comparisons

	$\left \mathbf{S}_{x}^{\omega_{0}}\right $	$ S_x^Q $
Passive RLC	$\leq \frac{1}{2}$	1,1/2
+KRC Equal R, Equal C (	κ=3-1/Q) 0,1/2	Q, 2Q, 3Q
Equal R, K=1 (C	,	0,1/2, 2Q <sup>2</sup>
Bridged-T Feedback	0,1/2	1/3,1/2, 1/6
Two-Integrator Loop	0,1/2	1,1/2, 0
-KRC	less than or equal to	1/2 less than or equal to 1/2

Substantial Differences Between (or in) Architectures

How do active sensitivities compare?

$$S_{\infty}^{\pm} = ?$$
  $S_{\pm}^{\pm} = ?$ 

Recall 
$$S_{x}^{f} = \frac{\partial f}{\partial x} \frac{x}{f}$$

So  $\frac{\partial f}{\partial x} = \frac{\partial x}{\partial x} S_{x}^{f}$ 

but if X is ideally O, not useful

$$\frac{t}{\sqrt{2t}} = \sqrt{2t}$$

$$\sqrt{2t} = \sqrt{2t}$$

## Where we are at with sensitivity analysis:

### Considered a group of five second-order filters

#### Passive Sensitivity Analysis

- Closed form expressions were obtained for  $\omega_0$  and Q
- Tedious but straightforward calculations provided passive sensitivities directly from the closed form expressions ???

#### **Active Sensitivity Analysis**

• Closed form expressions for  $\omega_0$  and Q are very difficult or impossible to obtain

### If we consider higher-order filters

#### Passive Sensitivity Analysis

• Closed form expressions for  $\omega_0$  and Q are very difficult or impossible to obtain for many useful structures

#### **Active Sensitivity Analysis**

• Closed form expressions for  $\omega_0$  and Q are very difficult or impossible to obtain

Need some better method for obtaining sensitivities when closed-form expressions are difficult or impractical to obtain or manipulate!!

# Relationship between pole sensitivities and $\omega_0$ and Q sensitivities

$$p = -\alpha + j\beta$$

$$D_{2}(s) = (s-p)(s-p^{*})$$

$$D_{2}(s) = (s+\alpha - j\beta)(s+\alpha + j\beta)$$

$$D_{2}(s) = s^{2} + s(2\alpha) + (\alpha^{2} + \beta^{2})$$

$$D_{2}(s) = s^{2} + s\frac{\omega_{0}}{Q} + \omega_{0}^{2}$$

$$P^{*} \times$$

# Relationship between active pole sensitivities and $\omega_0$ and Q sensitivities

Define 
$$D(s)=D_0(s)+t D_1(s)$$
 (from bilinear form of  $T(s)$ )

Recall: 
$$s_{\tau}^{p} = \frac{-D_{1}(p)}{\frac{\partial D(s)}{\partial s}\Big|_{s=p,\tau=0}}$$

Theorem: 
$$\Delta p \cong \tau s_r^r$$

Theorem: 
$$\Delta \alpha \cong \tau \operatorname{Re}(s_{\tau}^{r})$$

$$\Delta \beta \cong \tau \operatorname{Im}(s^{r})$$

#### Theorem:

$$\frac{\Delta\omega_0}{\omega_0} \cong \frac{1}{2Q} \frac{\Delta\alpha}{\omega_0} + \sqrt{1 - \frac{1}{4Q^2}} \frac{\Delta\beta}{\omega_0} \qquad \qquad \frac{\Delta Q}{Q} \cong -2Q \left(1 - \frac{1}{4Q^2}\right) \frac{\Delta\alpha}{\omega_0} + \sqrt{1 - \frac{1}{4Q^2}} \frac{\Delta\beta}{\omega_0}$$

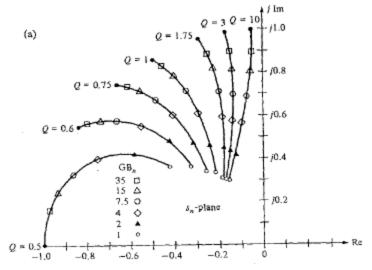
Claim: These theorems, with straightforward modification, also apply to other parameters (R, C, L, K, ...) where,  $D_0(s)$  and  $D_1(s)$  will change since the parameter is different

#### +KRC

$$\begin{split} & \underbrace{\frac{V_{\bullet}}{V_{\bullet}} = \frac{1}{RC}, \quad Q = \frac{1}{3 - K_{\bullet}}}_{S^{2} + s} \frac{1}{Q} \omega_{\bullet}^{2} - \frac{1}{Q} \omega_{\bullet}^{2}}_{S^{2} + s} \frac{\left(3 - \frac{1}{Q}\right) \omega_{\bullet}^{2}}{GB} + \omega_{\bullet}^{2} + \frac{\left(3 - \frac{1}{Q}\right)}{GB} s(s^{2} + s) \omega_{\bullet} + \omega_{\bullet}^{2}} - \frac{\Delta \omega}{\sqrt{1 - \frac{1}{2Q^{3}}}} \frac{\omega_{\bullet}}{GB} - \frac{\Delta \beta}{GB} = -\frac{1}{2} \left(3 - \frac{1}{Q}\right)^{2} \frac{\left(1 - \frac{1}{2Q^{3}}\right)}{\sqrt{1 - \frac{1}{4Q^{3}}}} \frac{\omega_{\bullet}}{GB} - \frac{\Delta \omega_{\bullet}}{\omega_{\bullet}} = -\frac{1}{2} \left(3 - \frac{1}{Q}\right)^{2} \frac{\omega_{\bullet}}{GB} - \frac{\Delta \omega}{GB} - \frac{1}{2} \left(3 - \frac{1}{Q}\right)^{2} \frac{\omega_{\bullet}}{GB} - \frac{\Delta \omega}{GB} - \frac{1}{2} \left(3 - \frac{1}{Q}\right)^{2} \frac{\omega_{\bullet}}{GB} - \frac{\Delta \omega_{\bullet}}{GB} - \frac{1}{2} \sqrt{\frac{C_{1}}{C_{2}}} - \frac{V_{\bullet}}{V_{1}} = \frac{\omega_{\bullet}^{2}}{s^{2} + s} \frac{\omega_{\bullet}}{Q} + \omega_{\bullet}^{2} + \frac{s}{GB} \left[s^{3} + s\omega_{\bullet}\left(2Q + \frac{1}{Q}\right) + \omega_{\bullet}^{2}\right] - \frac{\Delta \omega}{GB} - \frac{\Delta \omega}{GB}, \quad \frac{\Delta \beta}{\omega_{\bullet}} = -Q \frac{\left(1 - \frac{1}{2Q^{3}}\right)}{\sqrt{1 - \frac{1}{4Q^{2}}}} \frac{\omega_{\bullet}}{GB} - \frac{\Delta \omega_{\bullet}}{GB} - \frac{\Delta \omega}{GB}, \quad \frac{\Delta \beta}{Q} = Q \frac{\omega_{\bullet}}{GB} - \frac{\Delta \omega}{GB} - \frac{\Delta \omega}{GB}, \quad \frac{\Delta Q}{Q} = Q \frac{\omega_{\bullet}}{GB} - \frac{\Delta \omega}{GB} - \frac{\Delta \omega}{GB}, \quad \frac{\Delta Q}{Q} = Q \frac{\omega_{\bullet}}{GB} - \frac{\Delta \omega}{GB} - \frac{\Delta \omega}{GB}, \quad \frac{\Delta Q}{Q} = Q \frac{\omega_{\bullet}}{GB} - \frac{\Delta \omega}{GB} - \frac{\Delta \omega$$

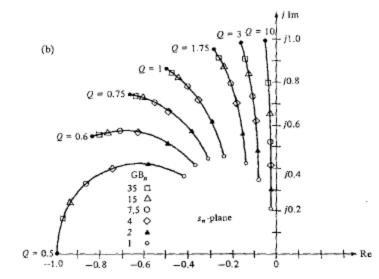
where 
$$s_n = \frac{s}{\omega_n}$$
,  $GB_n = \frac{GB}{\omega_n}$ .





√Fig. 10-5a Plot of upper half-plane root of

$$s_s^3 + s_s^2 \left(3 + \frac{QGB_s}{3Q - 1}\right) + s_s \left(1 + \frac{GB_s}{3Q - 1}\right) + \frac{QGB_s}{3Q - 1} = 0$$
 (Equal-R, equal-C)



◆Fig. 10-5b Plot of upper half plane root of

$$\underbrace{S_{\bullet}^{1} + s_{\bullet}^{2} \left(2Q + \frac{1}{Q} + GB_{\bullet}\right) + s_{\bullet} \left(1 + \frac{GB_{\bullet}}{Q}\right) + GB_{\bullet} = 0}_{\text{(Unity-gain, equal-}R)}$$

#### Bridged T Feedback

Table 10-3 Infinite-gain Realization (see Fig. 10-10b)

#### Equal-R

$$\begin{split} & \omega_* = \frac{1}{R\sqrt{C_1}} \frac{1}{C_2}; \qquad Q = \frac{1}{3} \sqrt{\frac{C_1}{C_3}} \\ & \frac{V_*}{V_t} = -\frac{\omega_e^2}{x^2 + s \frac{\omega_e}{Q} + \omega_e^2 + \frac{s}{GB} \left[ s^2 + s \omega_e \left( 3Q + \frac{1}{Q} \right) + 2 \omega_e^2 \right]} \\ & - \frac{\Delta \alpha}{\omega_e} \approx \frac{\omega_e}{GB}, \qquad \frac{\Delta \beta}{\omega_e} \approx -\frac{1}{2} \frac{3Q - \frac{1}{Q}}{\sqrt{1 - \frac{1}{4Q^2}}} \frac{\omega_e}{GB} \\ & \frac{\Delta \omega_e}{\omega_e} \approx -\frac{3Q}{2} \frac{\omega_e}{GB}, \qquad \frac{\Delta Q}{Q} \approx \frac{Q}{2} \frac{\omega_e}{GB} \end{split}$$

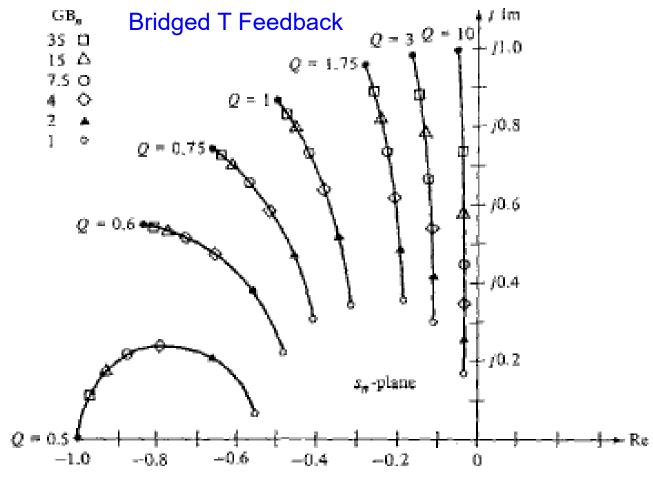


Fig. 10-12 Plot of upper half-plane root of

$$s_{s}^{3} + s_{s}^{2} \left(3Q + \frac{1}{Q} + GB_{s}\right) + s_{s} \left(2 + \frac{GB_{s}}{Q}\right) + GB_{s} = 0$$

#### Two integrator loop architecture

Equal-R (except  $R_Q$ ) and Equal-C

$$\begin{split} & \omega_* = \frac{1}{RC}, \qquad Q = \frac{R_0}{R} \\ & \frac{V_s}{V_1} \cong \frac{\omega_s^3 \left(\frac{2}{GB} s + 1\right)}{s^2 + s \frac{\omega_s}{Q} + \omega_s^3 + \frac{1}{GB} \left(4s \left[s^2 + s \omega_s \left(\frac{1}{2} + \frac{1}{Q}\right) + \frac{\omega_s^2}{4Q}\right]\right)} \\ & - \frac{\Delta \sigma}{\omega_s} \cong 2 \left(1 + \frac{1}{4Q}\right) \frac{\omega_s}{GB}, \qquad \frac{\Delta \beta}{\omega_s} \cong - \frac{\left(1 - \frac{1}{Q} - \frac{1}{4Q^2}\right)}{\sqrt{1 - \frac{1}{4Q^2}}} \frac{\omega_s}{GB} \end{split}$$

$$\frac{\Delta \omega_s}{\omega_s} \simeq -\frac{\omega_s}{GB}, \quad \frac{\Delta Q}{Q} \simeq 4Q \frac{\omega_s}{GE}$$

#### Two integrator loop architecture

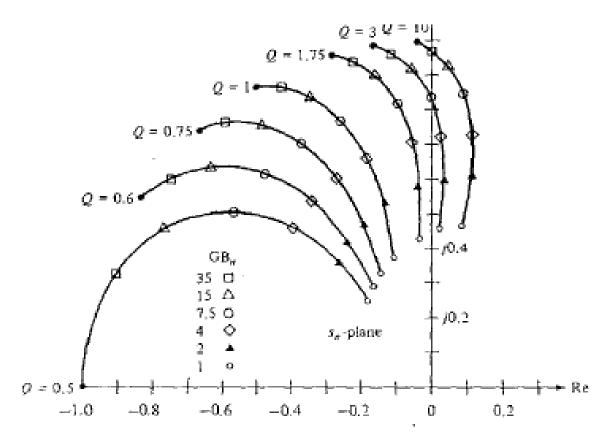


Fig. 10-17 Plot of upper half-plane root of

$$s_n^2 + s_n^2 \left( \frac{1}{2} + \frac{1}{Q} + \frac{GB_n}{4} \right) + s_n \frac{1}{4Q} \left( 1 + GB_n \right) + \frac{GB_n}{4} = 0$$

#### - KRC

## Equal-R, Equal-C

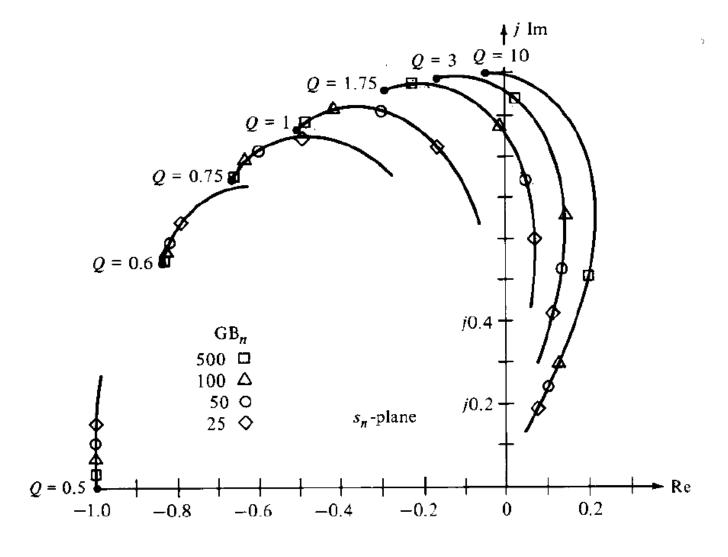
$$\omega_o = \frac{\sqrt{5 + K_o}}{RC}, \qquad Q = \frac{\sqrt{5 + K_o}}{5}$$

$$\frac{V_o}{V_i} = -\frac{\omega_o^2 \left(1 - \frac{1}{5Q^2}\right)}{s^2 + s\frac{\omega_o}{Q} + \omega_o^2 + \frac{s}{GB} \left[s^2(25Q^2 - 4) + s\omega_o \left(20Q - \frac{3}{Q}\right) + \left(2 - \frac{1}{5Q^2}\right)\omega_o^2\right]}{\left(\omega_a \leqslant \frac{\omega_o}{2Q}\right)}$$

$$-\frac{\Delta \alpha}{\omega_{o}} \cong \frac{25Q^{2}}{2} \left(1 - \frac{1}{5Q^{2}}\right) \left(1 - \frac{6}{25Q^{2}}\right) \frac{\omega_{o}}{GB}, \qquad \frac{\Delta \beta}{\omega_{o}} \cong \frac{35Q}{4} \frac{\left(1 - \frac{1}{5Q^{2}}\right) \left(1 - \frac{6}{35Q^{2}}\right)}{\sqrt{1 - \frac{1}{4Q^{2}}}} \frac{\omega_{o}}{GB}$$

$$\frac{\Delta\omega_o}{\omega_o} \cong \frac{5Q}{2} \left( 1 - \frac{1}{5Q^2} \right) \frac{\omega_o}{GB}, \qquad \frac{\Delta Q}{Q} \cong 25Q^3 \left( 1 - \frac{1}{5Q^2} \right) \left( 1 - \frac{7}{5Q^2} \right) \frac{\omega_o}{GB}$$

- KRC



## **Active Sensitivity Comparisons**

	$\Delta\omega_0$	$\Delta Q$
	$\omega_0$	Q
Passive RLC	NA	NA
+KRC		1/ 1/2
Equal R, Equal C (K=3-1/Q)	$-\frac{1}{2}\left(3-\frac{1}{Q}\right)^2\tau\omega_0$	$-\frac{1}{2}\left(3-\frac{1}{Q}\right)^2\tau\omega_0$
Equal R, K=1 $(C_1=4Q^2C_2)$	$-Q\tau\omega_0$	$Q \tau \omega_0$
Bridged-T Feedback	$-\frac{3}{2}Q\tau\omega_0$	$\frac{1}{2}Q\tau\omega_0$
Two-Integrator Loop	$-\tau\omega_0$	$4Q\tau\omega_0$
-KRC	$\frac{5}{2}$ Q $\tau\omega_0$	$25Q^3\tau\omega_0$

**Substantial Differences Between Architectures** 

## Are these passive sensitivities acceptable?

	$ S_{x}^{\omega_0} $	$ S_x^Q $
Passive RLC	$\leq \frac{1}{2}$	1,1/2
+KRC		
Equal R, Equal C (F	<=3-1/Q) 0,1/2	Q, 2Q, 3Q
Equal R, K=1 (C.	$_{1}$ =4Q $^{2}$ C $_{2}$ ) 0,1/2	0,1/2, 2Q <sup>2</sup>
Bridged-T Feedback		
	0,1/2	1/3,1/2, 1/6
Two-Integrator Loop	0,1/2	1,1/2, 0
-KRC	less than or equal to 1/2	less than or equal to 1/

## Are these active sensitivities acceptable?

# Active Sensitivity Comparisons

Passive RLC	$\frac{\Delta\omega_0}{\omega_0}$	$\frac{\Delta Q}{Q}$
+KRC		2
Equal R, Equal C (K=3-1/Q)	$-\frac{1}{2}\left(3-\frac{1}{Q}\right)^2\tau\omega_0$	$-\frac{1}{2}\left(3-\frac{1}{Q}\right)^2\tau\omega_0$
Equal R, K=1 $(C_1=4Q^2C_2)$	$-Q\tau\omega_0$	$Q\tau\omega_0$
Bridged-T Feedback	$-\frac{3}{2}Q\tau\omega_0$	$\frac{1}{2}Q\tau\omega_0$
Two-Integrator Loop	$-\tau\omega_0$	$4Q\tau\omega_0$
-KRC	$\frac{5}{2}$ Q $\tau\omega_0$	$25Q^3\tau\omega_0$

#### Are these sensitivities acceptable?

#### **Passive Sensitivities:**

$$\frac{\Delta\omega_{_{0}}}{\omega_{_{0}}}\cong S_{_{x}}^{\omega_{_{0}}}\frac{\Delta x}{x}$$

In integrated circuits,  $\Delta$  R/R and  $\Delta$  C/C due to process variations can be  $\pm$  30% or larger due to process variations

Many applications require  $\Delta\omega_0/\omega_0$ <.001 or smaller and similar requirements on  $\Delta Q/Q$ 

Even if sensitivity is around ½ or 1, variability is often orders of magnitude too large

#### **Active Sensitivities:**

All are proportional to  $\tau\omega_0$ 

Some architectures much more sensitive than others

Can reduce  $\tau\omega_0$  by making GB large but this is at the expense of increased power and even if power is not of concern, process presents fundamental limits on how large GB can be made

Observe that for the +KRC circuit, the somewhat arbitrary use of the DOF has a major impact on performance

Similar observations can be made for other structures

$$\frac{\Delta \omega_0}{\omega_0} \qquad \frac{\Delta Q}{Q}$$
 Equal R, Equal C (K=3-1/Q) 
$$-\frac{1}{2} \left( 3 - \frac{1}{Q} \right)^2 \tau \omega_0 \qquad -\frac{1}{2} \left( 3 - \frac{1}{Q} \right)^2 \tau \omega_0$$
 Equal R, K=1 (C<sub>1</sub>=4Q<sup>2</sup>C<sub>2</sub>) 
$$-Q \tau \omega_0 \qquad Q \tau \omega_0$$
 
$$-\frac{1}{2} \left( 3 - \frac{1}{Q} \right)^2 \tau \omega_0 \qquad -\frac{1}{2} \left( 3 - \frac{1}{Q} \right)^2 \tau \omega_0$$
 Equal R, K=1 (C<sub>1</sub>=4Q<sup>2</sup>C<sub>2</sub>) 
$$-Q \tau \omega_0 \qquad Q \tau \omega_0$$

Challenge: Can a major improvement in performance be obtained by a more judicious use of the two free DOF for the +KRC circuit?

## 1. Predistortion

Design circuit so that <u>after</u> component shift, correct pole locations are obtained

Predistortion is generally used in integrated circuits to remove the bias associated with inadequate amplifier bandwidth

Predistortion does not help with process variations of passive components

Tedious process after fabrication since depends on individual components

Temperature dependence may not track

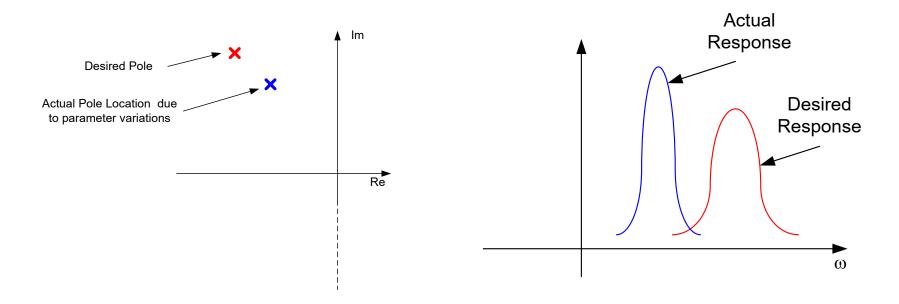
Difficult to maintain over time and temperature

Over-ordering will adversely affect performance

Seldom will predistortion alone be adequate to obtain acceptable performance Bell Labs did to this in high-volume production (STAR Biquad)

## Predistortion

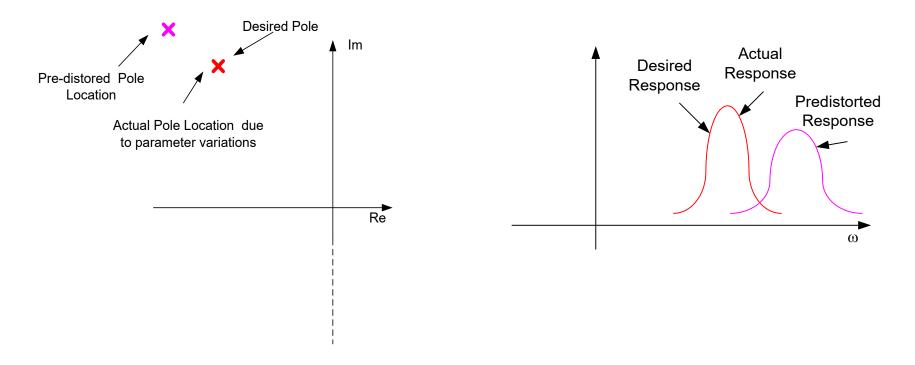
Design circuit so that <u>after</u> component shift, correct pole locations are obtained



Pole shift due to parametric variations (e.g. inadequate GB)

## Predistortion

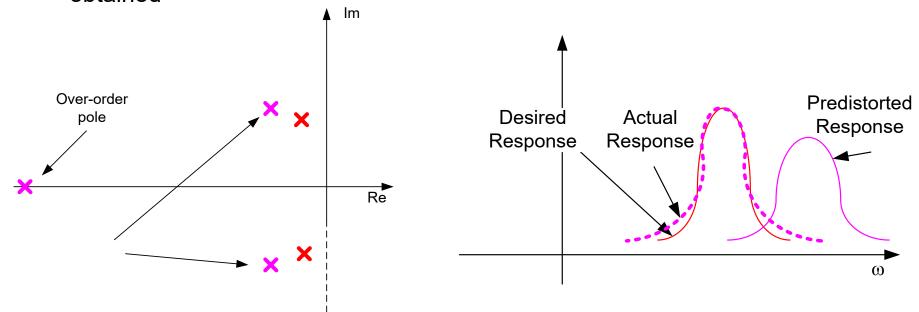
Design circuit so that <u>after</u> component shift, correct pole locations are obtained



Pre-distortion concept

## Predistortion

Design circuit so that <u>after</u> component shift, correct pole locations are obtained



Over-ordering Limitations with Pre-distortion
Parasitic Pole Affects Response

Predistortion almost always done even if benefits only modest

Not effective if significant deviations exist before predistortion

## Trimming

#### a) Functional Trimming

- trim parameters of actual filter based upon measurements
- difficult to implement in many structures
- manageable for cascaded biquads

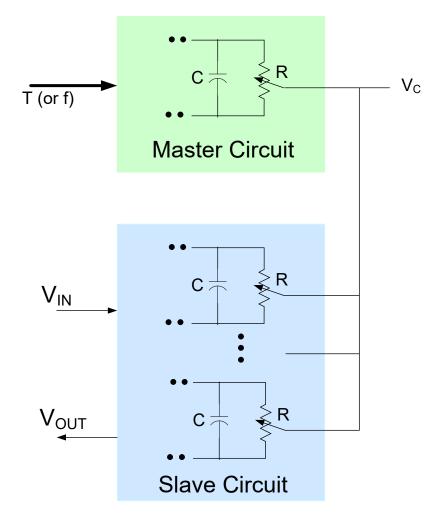
#### b) Deterministic Trimming (much preferred)

- Trim component values to their ideal value
  - Continuous-trims of resistors possible in some special processes
  - Continuous-trim of capacitors is more challenging
  - Link trimming of Rs or Cs is possible with either metal or switches
- If all components are ideal, the filter should also be ideal
  - R-trimming algorithms easy to implement
  - Limited to unidirectional trim
  - Trim generally done at wafer level for laser trimming, package for link trims
- Filter shifts occur due to stress in packaging and heat cycling

#### c) Master-slave reference control (depends upon matching in a process)

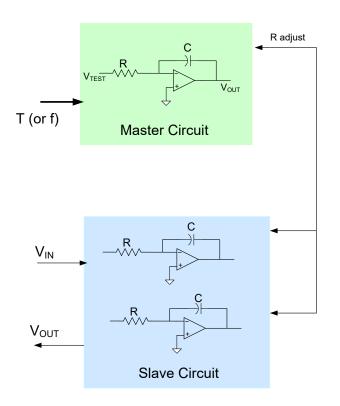
- Can be implemented in discrete or integrated structures
- Master typically frequency or period referenced
- Most effective in integrated form since good matching possible
- Widely used in integrated form

#### Master-slave Control (depends upon matching in a process)



- Automatically adjust R (or C) in the Master Circuit to match RC to T
- Rely on matching to match RC products in Slave Circuit to T
- Matching can be very good (1% or 0.1% or better)
- But does nothing to compensate for local random variations

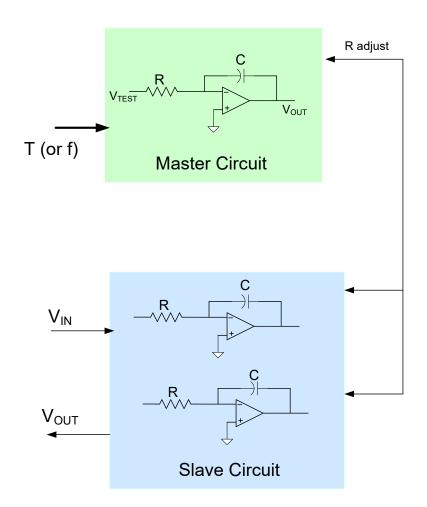
#### **Master-slave Example:**



$$T(s) = \frac{V_{OUT}}{V_{TEST}} = -\frac{1}{RCs}$$

- Key parameter of integrator is unity gain frequency I<sub>0</sub>=1/RC
- Adjust R in Master Circuit so that I<sub>0</sub>=1 at the input frequency f
- With matching, unity gain frequency of all integrators in Slave Circuit will also be 1
- May require considerable overhead to trim circuit elements
- Compensates for combined component variations and BW limitations

#### **Master-slave Example:**



$$T(s) = \frac{V_{OUT}}{V_{TEST}} = -\frac{1}{RCs}$$

$$T_{ACT}(s) = \frac{V_{OUT}}{V_{TEST}} = -\frac{1}{RCs + \tau(s + RCs^2)}$$

- Over-ordering will limit accuracy of master-slave approach even if unity gain frequency of master circuit is precisely obtained
- Technique is often used to maintain good control of effective RC products
- Power and area overhead but Master circuit may be off most of the time to reduce power overhead

What can be done to address these problems?

#### 3. Select Appropriate Architecture

Helps a lot

Best architectures are not known

Performance of good architectures often not good enough

What can be done to address these problems?

# 4. Different Approach for Filter Implementation

- Frequency Referenced Filters
   Switched-Capacitor Filters
- DSP- Based Filter Implementation
- Other Niche Methods

#### Summary of Sensitivity Observations

- Sensitivity varies substantially from one implementation to another
- Variability too high, even with low sensitivity, for more demanding applications
- Methods of managing high variability
  - Select good structures
  - Trimming

**Functional** 

**Deterministic** 

Predistortion

In particular, for active sensitivities

Useful but not a total solution

Frequency Referenced Techniques

Master-Slave Control

Depends upon matching

Can self-trim or self-compensate

**Switched-Capacitor Filters** 

AD/digital filter/D/A

Alternate Design Approach

Other methods

## Filter Design Process

### Establish Specifications

- possibly  $T_D(s)$  or  $H_D(z)$
- magnitude and phase characteristics or restrictions
- time domain requirements

#### Approximation

- obtain acceptable transfer functions  $T_A(s)$  or  $H_A(z)$
- possibly acceptable realizable time-domain responses

#### Where we are at

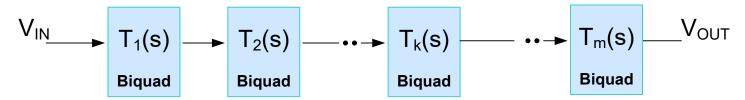
#### **Synthesis**

- build circuit or implement algorithm that has response close to  $T_A(s)$  or  $H_A(z)$
- actually realize  $T_R(s)$  or  $H_R(z)$



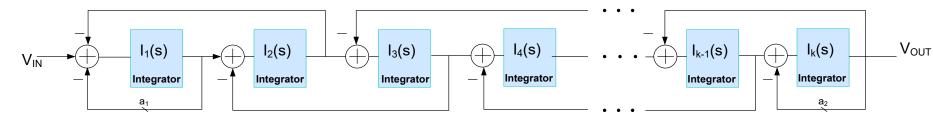
#### Most designs today use one of the following three basic architectures

#### **Cascaded Biquads**

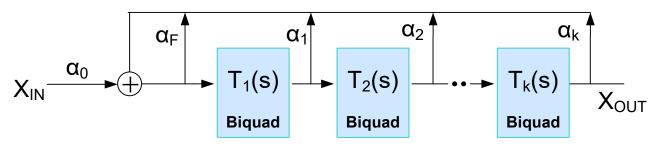


$$T(s) = T_1 T_2 \cdot \cdot T_m$$

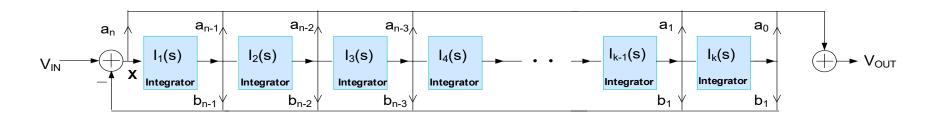
#### Leapfrog



#### Multiple-loop Feedback – One type shown (less popular)



#### Multiple-loop Feedback – Another type



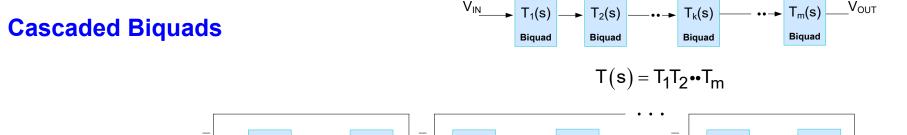
$$X = V_{IN} - X \bullet \sum_{k=1}^{n} b_{n-k} \left(\frac{I_0}{s}\right)^k$$

$$V_{OUT} = X \bullet \sum_{k=0}^{n} a_{n-k} \left(\frac{I_0}{s}\right)^k$$

$$T(s) = \frac{\sum_{k=0}^{n} a_{n-k} \left(\frac{I_0}{s}\right)^k}{1 + \sum_{k=1}^{n} b_{n-k} \left(\frac{I_0}{s}\right)^k}$$

$$T(s) = \frac{\sum_{k=0}^{n} a_{n-k} I_0^k s^{n-k}}{s^n + \sum_{k=1}^{n} b_{n-k} I_0^k s^{n-k}}$$

- Termed the direct synthesis method
- Directly implements the coefficients in the numerator and denominator
- Approach followed in the Analog Computers
- Not particularly attractive from an overall performance viewpoint



Multiple-loop Feedback – One type shown  $X_{IN} \xrightarrow{\alpha_0} T_1(s) \xrightarrow{\alpha_1} T_2(s) \xrightarrow{\alpha_2} T_k(s) \xrightarrow{\alpha_1} T_k(s) \xrightarrow{\alpha_0} T_$ 

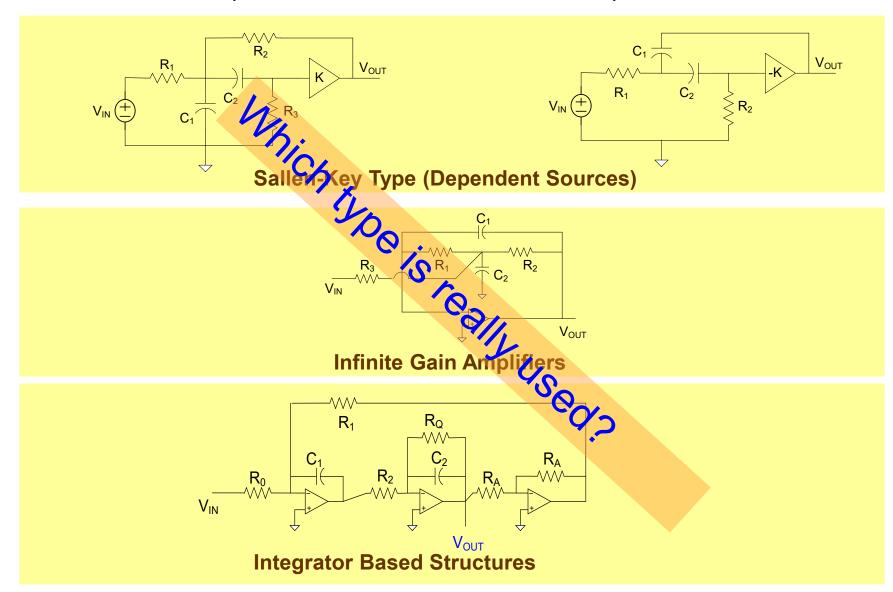
Will study details of all three types of architectures later

Observation: All filters are comprised of summers, biquads and integrators

Consider now the biquads

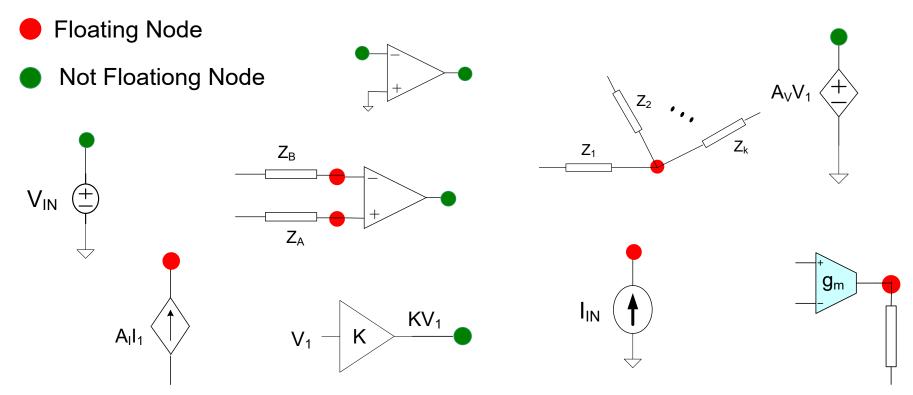
### Biquad Filters Design Considerations

Several different Biquads were considered and other implementations exist

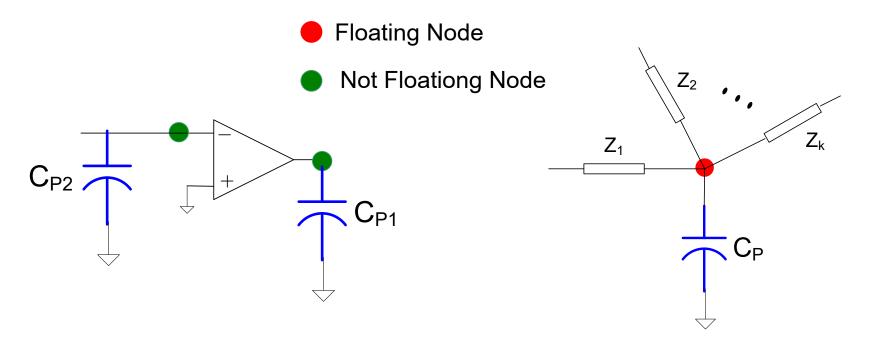


## Floating Nodes

A node in a circuit is termed a floating node if it is not an output node of a ground-referenced voltage-output amplifier (dependent or independent), not connected to a ground-referenced voltage source, or not connected to a ground-referenced null-port



### Parasitic Capacitances on Floating Nodes



Parasitic capacitances ideally have no affect on filter when on a non-floating node but directly affect transfer function when they appear on a floating node

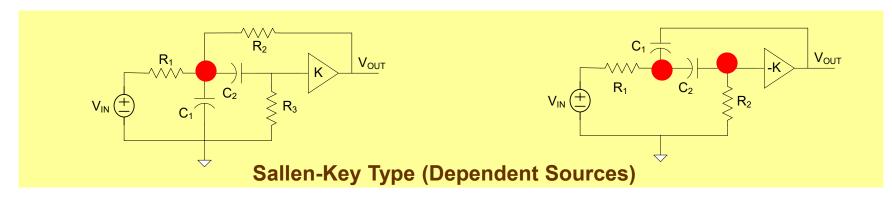
Parasitic capacitances are invariably large, nonlinear, and highly process dependent in integrated filters. Thus, it is difficult to build accurate integrated filters if floating nodes are present

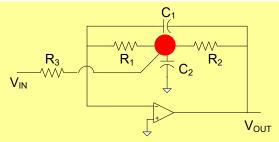
Generally avoid floating nodes, if possible, in integrated filters

### Which type of Biquad is really used?

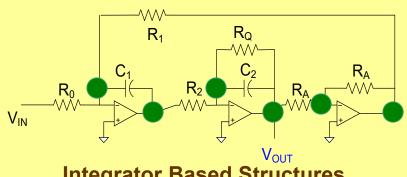
Not Floationg Node

Floating Node



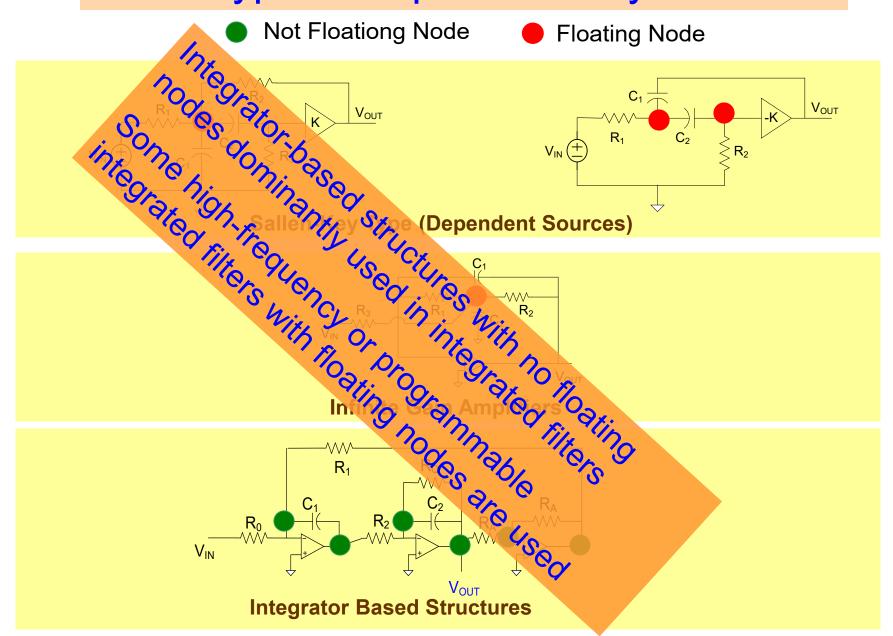


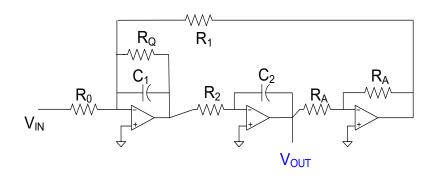
#### **Infinite Gain Amplifiers**

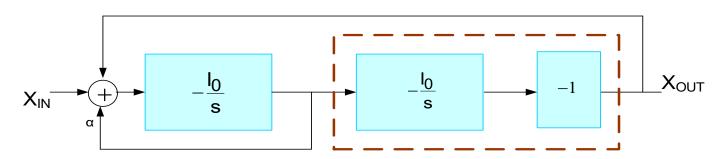


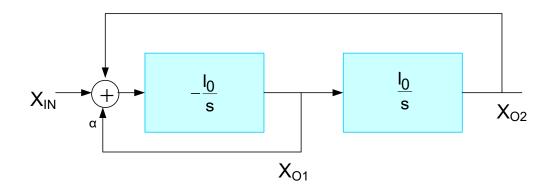
**Integrator Based Structures** 

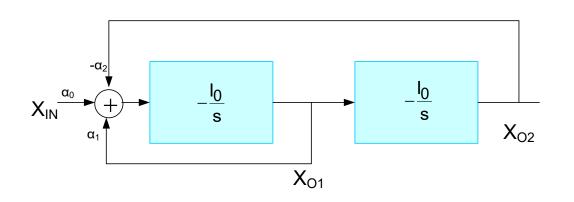
### Which type of Biquad is really used?



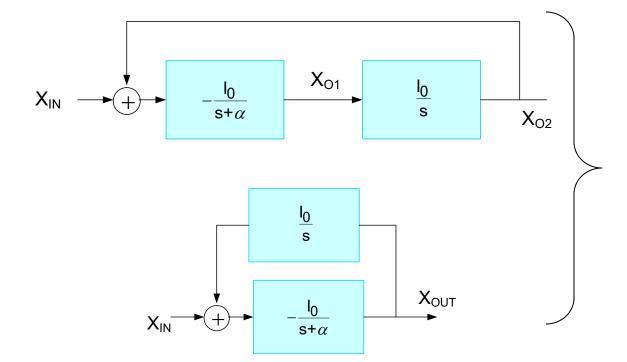




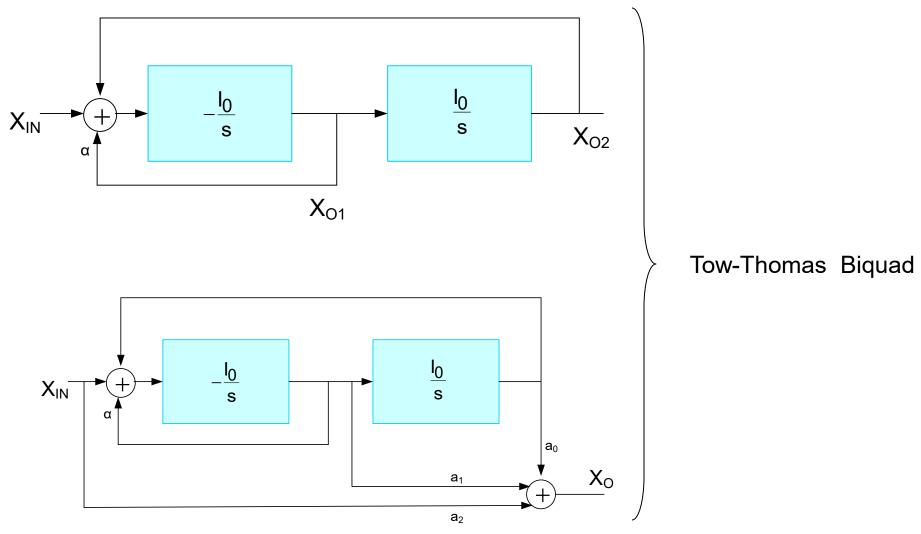




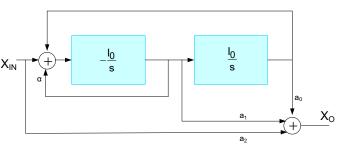
State Variable Biquad (Alt KHN Biquad)

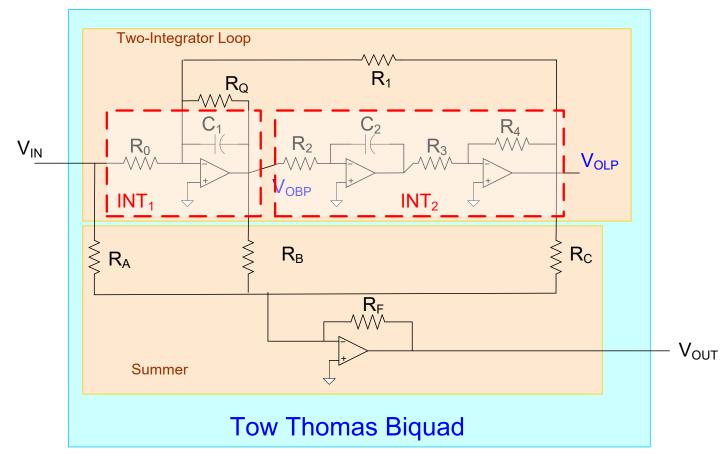


Integrator and lossy integrator in a loop



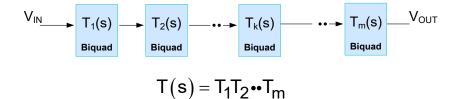
With arbitrary zero locations



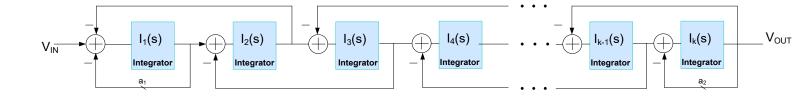


- Integrator-based biquads all involve two integrators in a loop
- All integrator-based biquads discussed have no floating nodes
- Most biquads in integrated filters are based upon two integrator loop structures
- The summers are usually included as summing inputs on the integrators
- The loss can be combined with the integrator to form a lossy integrator
- Performance of the minor variants of the two integrator loop structures are comparable

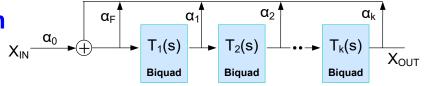
**Cascaded Biquads** 



Leapfrog



Multiple-loop Feedback - One type shown



Observation: All filters are comprised of summers, biquads and integrators

And biquads usually made with summers and integrators

Integrated filter design generally focused on design of integrators, summers, and amplifiers (Op Amps)

Will now focus on the design of integrators, summers, and op amps

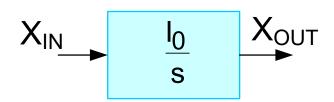
## Basic Filter Building Blocks

(particularly for integrated filters)

- IntegratorsSummers

  - Operational Amplifiers

### Integrator Characteristics of Interest



$$I(s) = \frac{I_0}{s}$$

Properties of an ideal integrator:

$$|I(j\omega)| = \frac{I_0}{\omega}$$

Gain decreases with 1/ω

$$\angle I(j\omega) = -90^{\circ}$$

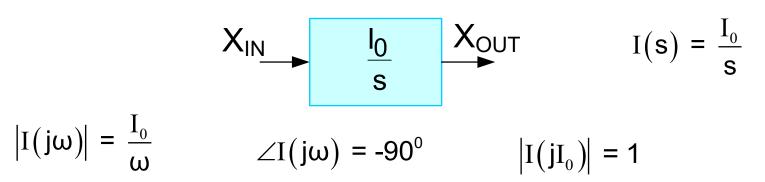
Phase is a constant -90°

$$\left|I(jI_0)\right| = 1$$

Unity Gain Frequency =  $I_0$ 

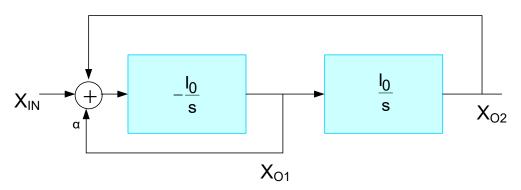
How important is it that an integrator have all 3 of these properties?

### Integrator Characteristics of Interest



How important is it that an integrator have all 3 of these properties?

#### Consider a filter example:



$$T(s) = \frac{-I_0^2}{s^2 + \alpha I_0 s + I_0^2}$$

$$Q = \frac{1}{s^2} \qquad \omega_0 = I_0$$

Band edges proportional to I<sub>0</sub> Phase critical to make Q expression valid

In many (most) applications it is critical that an integrator be very nearly ideal

(in the frequency range of interest)

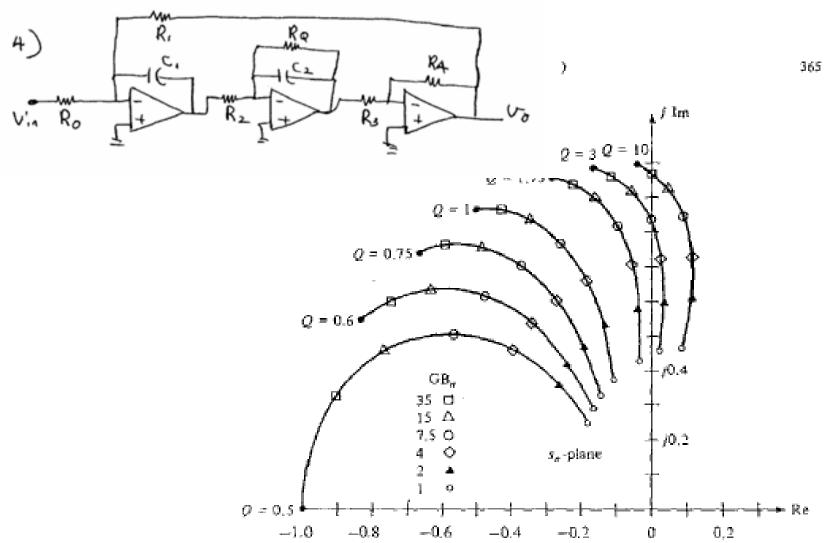
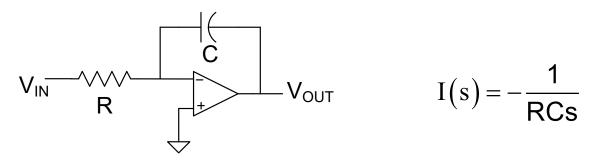


Fig. 10-17 Plot of upper half-plane root of

$$s_n^2 + s_n^2 \left( \frac{1}{2} + \frac{1}{Q} + \frac{GB_n}{4} \right) + s_n \frac{1}{4Q} \left( 1 + GB_n \right) + \frac{GB_n}{4} = 0$$

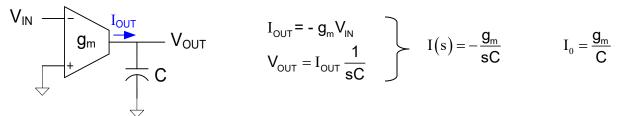


$$I(s) = -\frac{1}{RCs}$$

$$I_0 = \frac{1}{RC}$$

Inverting Active RC Integrator

Are there other integrator structures?



$$I_{OUT} = -g_{m}V_{IN}$$

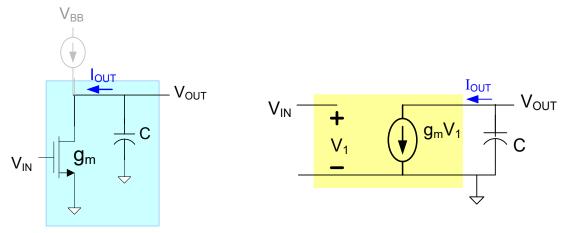
$$V_{OUT} = I_{OUT} \frac{1}{sC}$$

$$I(s) = -\frac{g_{m}}{sC}$$

$$\boldsymbol{I}_0 = \frac{\boldsymbol{g}_m}{\boldsymbol{C}}$$

Termed an OTA-C or a gm-C integrator

Are there other integrator structures?



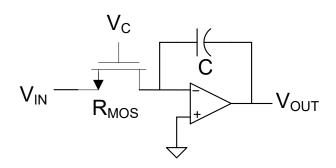
$$I_{OUT} = g_{m}V_{IN}$$

$$V_{OUT} = -I_{OUT} \frac{1}{sC}$$

$$I(s) = -\frac{g_{m}}{sC}$$

Termed a TA-C integrator

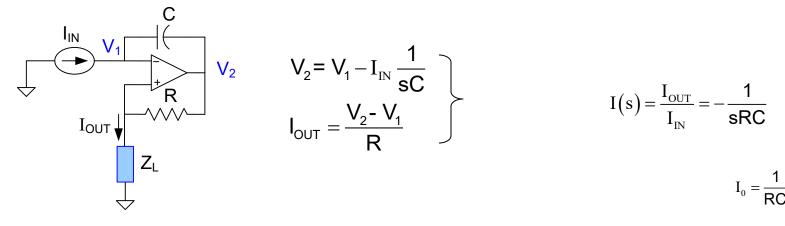
$$I_0 = \frac{g_m}{C}$$



$$I(s) = -\frac{1}{sCR_{MOS}}$$

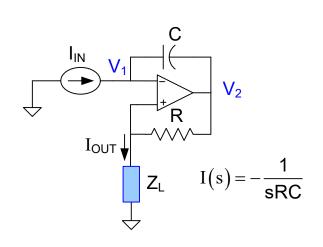
$$I_0 = -\frac{1}{R_{FET}C}$$

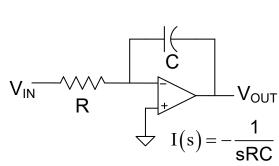
Are there other integrator structures?

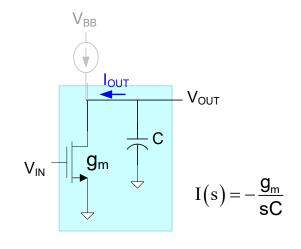


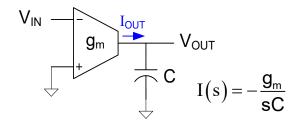
- Output current is independent of Z<sub>L</sub>
- Thus output impedance is ∞ so provides current output

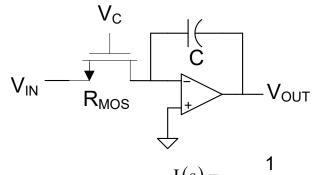
Termed active RC current-mode integrator







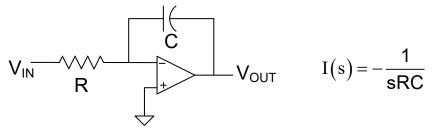




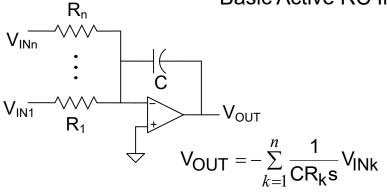
There are other useful integrator structures (some will be introduced later)

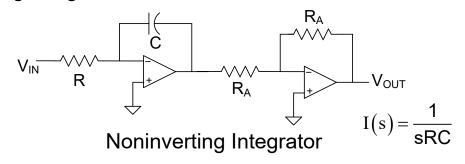
There are many different ways to build an inverting integrator

## Integrator Functionality

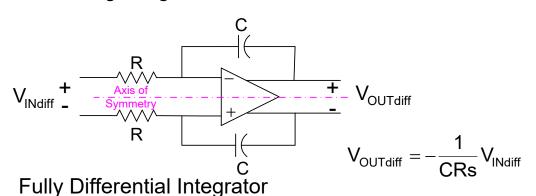


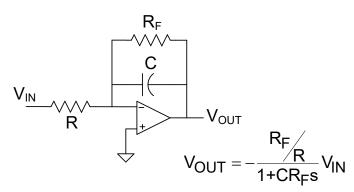
Basic Active RC Inverting Integrator





**Summing Integrator** 

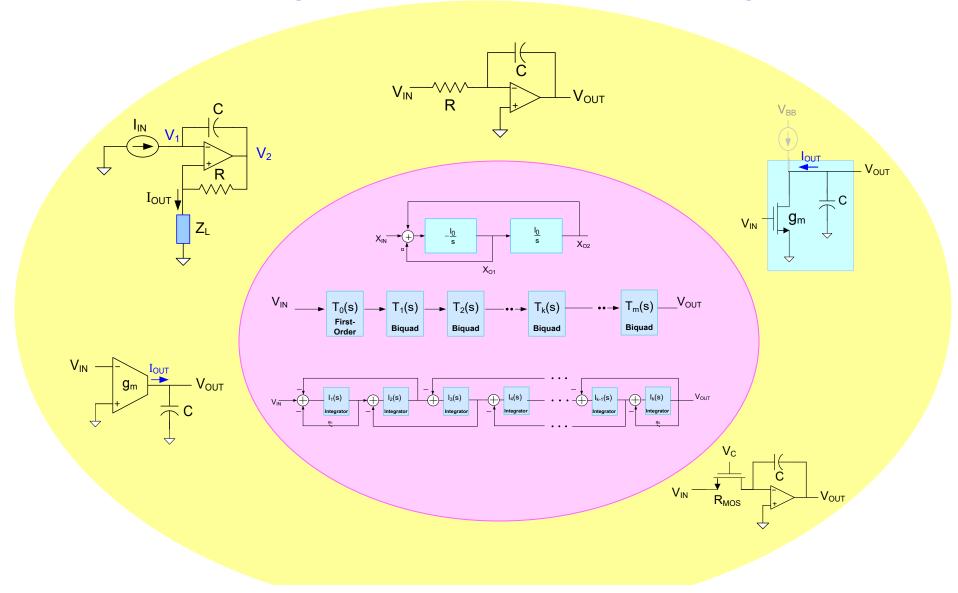




Lossy Integrator

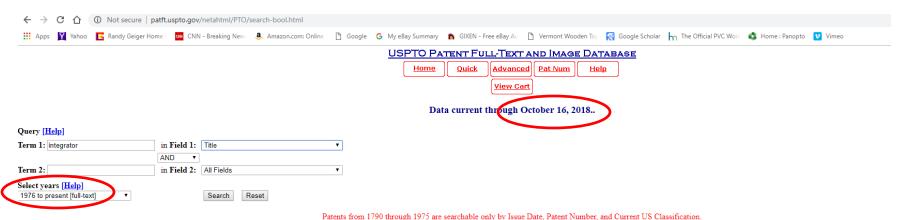
Many different types of functionality from basic inverting integrator Same modifications exist for other integrator architectures

### Integrator-Based Filter Design



Any of these different types of integrators can be used to build integrator-based filters

### Are new integrators still being invented?



When searching for specific numbers in the Patent Number field, utility patent numbers are entered as one to eight numbers in length, excluding commas (which are optional, as are leading zeroes).

Results of Search in US Patent Collection db for:
TTL/integrator: 351 patents.
Hits 1 through 50 out of 531

Next 50 Hits

Jump To

Refine Search TTL/integrator

#### Oct 16 2018

O	/Ct 10 20	
	PAT. NO.	Title
	1 10,082,922	Increasing the dynamic range of an integrator based mutual-capacitance measurement circuit
	2 10,074,004	Capacitive fingerprint sensor with integrator
	3 10,070,089	Inverting amplifier, integrator, sample hold circuit, ad converter, image sensor, and imaging apparatus
		■ Gated CDS integrator
	5 9,972,003	Pregame electronic commerce integrator
	6 9,954,514	Output range for interpolation architectures employing a cascaded integrator-comb (CIC) filter with a multiplier
	7 9,885,959	Illumination optical apparatus having deflecting member, lens, polarization member to set polarization in circumference direction, and optical integrator
	8 <u>9,885,872</u>	Illumination optical apparatus, exposure apparatus, and exposure method with optical integrator and polarization member that changes polarization state of light
	9 9,866,237	Low power switched capacitor integrator, analog-to-digital converter and switched capacitor amplifier
	10 <u>9,852,283</u>	Confirming the identity of integrator applications
Nov 2017	11 <u>9,825,646</u>	Integrator and A/D converter using the same
		System integrator and method for mapping dynamic COBOL constructs to object instances for the automatic integration to object-oriented computing systems
	13 <u>9,806,552</u>	Analog/digital converter with charge rebalanced integrator
	14 <u>9,800,256</u>	Semiconductor device including integrator and successive approximation register analog-to-digital converter and driving method of the same
	15 <u>9,753,559</u>	Feedback integrator current source, transistor, and resistor coupled to input
	16 <u>9,726,521</u>	Signal processing apparatus for processing time variant signal with first and second input signals comprising a weighting integrator, a magnitude detector and a gain-adjustable amplifier
	17 <u>9,709,242</u>	■ Shell integrator
		Projector having a rod integrator with an entrance plane smaller than an area light source
		Apparatus for overload recovery of an integrator in a sigma-delta modulator
		Increasing the dynamic range of an integrator based mutual-capacitance measurement circuit
		Integrator, AD converter, and radiation detection device
		■ Integrator, delta-sigma modulator, and communications device
	23 <u>9,628,103</u>	Multi-mode discrete-time delta-sigma modulator power optimization using split-integrator scheme
		Cascaded integrator-comb filter as a non-integer sample rate converter
		Electronic integrator for Rogowski coil sensors
		Shell integrator
		Sampling network and clocking scheme for a switched-capacitor integrator
		Confirming the identity of integrator applications
		Integrator and touch sensing system using the same
		System integrator and method for mapping dynamic COBOL constructs to object instances for the automatic integration to object-oriented computing systems
	31 <u>9,496,969</u>	Double integrator pulse wave shaper apparatus, system and method
Nov 2016	32 <u>9,495,563</u>	Analog integrator system and method
		Dynamic current source for amplifier integrator stages
		Low power and compact area digital integrator for a digital phase detector
		Integrator for class D audio amplifier
		Illumination system having first and second lens arrays including plano-convex lenses wherein some lenses in the second array include a first and a second lens element, projection-type display apparatus, and optical integrator
		Apparatuses, methods and systems for a universal payment integrator
		DC-DC converter controller apparatus with dual-counter digital integrator
		Charge balancing converter using a passive integrator circuit
		Delta-sigma modulator with reduced integrator requirements
		Compensation filter for cascaded-integrator-comb decimator
		System integrator and system integration method with reliability optimized integrated circuit chip selection
		Therapeutic integrator apparatus
		Increasing the dynamic range of an integrator based mutual-capacitance measurement circuit
		Active integrator for a capacitive sense array
		Current amplifier circuit, integrator, and ad converter
		Apparatuses and method of switched-capacitor integrator
		Systems and methods for preventing saturation of analog integrator output
		System integrator and method for mapping dynamic COBOL constructs to object instances for the automatic integration to object-oriented computing systems
1	50.0.130.006	Tong aided detection and disabling of integrator said up for aneed control in a vehicle

50 9,139,096 One-sided detection and disabling of integrator wind up for speed control in a vehicle

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51 9.063,789 Thybrid cloud integrator plug-in components
                 52 9.061,592 System and method for detecting power integrator malfunction
                 53 9.054,731 Integrator output swing reduction
                 54 9.039.190 Projector having integrator with greater illuminance in offset direction of projection lens and modulator
                 55 9.037.469 Automated communication integrator
                 56 9.014.322 Low power and compact area digital integrator for a digital phase detector
                 57 9,009,697 Hybrid cloud integrator
                 58 8.995.061 Speckle reduction using lenslet integrator
                 59 8,988,904 Power supply with integrator for controlling current
                 60 8.957.363 Differential photodiode integrator circuit for absorbance measurements
                 61 8.952.749 Filter with combined resonator and integrator
                 62 8.941,526 Time integrator and .DELTA..SIGMA. time-to-digital converter
                 63 8.937.567 Delta-sigma modulator, integrator, and wireless communication device
                 64 8,922,290 Pulse width modulator with two-way integrator
                 65 8.866.532 Passive integrator and method
                 66 8.866.531 Broadband analog radio-frequency integrator
                 67 8,860,491 Integrator output swing reduction technique for sigma-delta analog-to-digital converters
                 68 8.854.107 Integrator circuit with inverting integrator and non-inverting integrator
                 69 8.851.684 TOptical unit including an integrator optical system, and projection display device including the optical unit
                 70 8,835,827 Current integrator with wide dynamic range
                 71 8,824,626 Reduced-noise integrator, detector and CT circuits
                 72 8.816.763 T Integrator input error correction circuit and circuit method
                 73 8.779.831 Tintegrator
July 2014
                 74 8,775,003 Methods and systems for controlling a proportional integrator
                 75 8,767,343 T Disk drive increasing integrator output range to complete seek operation
                 76 8.724.080 Poptical raster element, optical integrator and illumination system of a microlithographic projection exposure apparatus
                 77 8,704,580 Circuit sharing time delay integrator
                 78 8.674.864 Integrator and oversampling A/D converter having the same
                 79 8.665.129 
☐ Complex second-order integrator and oversampling A/D converter having the same
                 80 8.659.343 Calibration for mixed-signal integrator architecture
                 81 8.653.867 Pulse modulated neural integrator circuit and associated phase locked loop
                 82 8.639.513 Automated communication integrator
                 83 8.638.420 Optical integrator, illuminating optical device, exposure apparatus and device manufacturing method
                 84 8,614,639 Integrator ramp generator with DAC and switched capacitors
                 85 8,611,013 Optical integrator, illumination optical device, aligner, and method for fabricating device
                 86 8.587.764 Optical integrator system, illumination optical apparatus, exposure apparatus, and device manufacturing method
                 87 8,575,988 Mixed-signal integrator architecture
                 88 8,573,779 Lighting device with plural light sources illuminating distinct regions of integrator
                 89 8,566.277 T System integrator and method for mapping dynamic COBOL constructs to object instances for the automatic integration to object-oriented computing systems
                 90 8,564,358 Integrator circuit with multiple time window functions
                 91 8,558,610 I Integrator input error correction circuit and circuit method
                 92 8.536.923 TIntegrator distortion correction circuit
                 93 8,526,487 Differential energy difference integrator
                 94 8,520,307 T Optical integrator for an illumination system of a microlithographic projection exposure apparatus
                 95 8.504.503 Pulse modulated neural integrator circuit
                 96 8.497.977 T Optical integrator, illumination optical system, exposure apparatus, and device manufacturing method
                 97 8,438,201 Digital fractional integrator
                 98 8.432.150 Methods for operating an array column integrator
                 99 8,432,149 Array column integrator
                 100 8.422.018 Optical measurement apparatus including hemispherical optical integrator
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#### Oct 16 2012 PAT. NO. Title 1 8,290,897 T System integrator and method for mapping dynamic COBOL constructs to object instances for the automatic integration to object-oriented computing systems 2 8,283,966 Integrator circuit 3 8,275,307 Vehicle audio integrator 4 8,264,388 Frequency integrator with digital phase error message for phase-locked loop applications 5 8,258,990 Integrator, resonator, and oversampling A/D converter 6 8,253,473 Integrated circuit of an integrator with enhanced stability and related stabilization method 7 8,199,038 Active resistance-capacitor integrator and continuous-time sigma-delta modulator with gain control function 8 8.164.873 Integrator and circuit-breaker having an integrator 9 8.145.597 T System integrator and method for mapping dynamic COBOL constructs to object instances for the automatic integration to object-oriented computing systems 10 8,129,972 T Single integrator sensorless current mode control for a switching power converter 11 8,125,262 Low power and low noise switched capacitor integrator with flexible input common mode range 12 8.098,377 Electric gated integrator detection method and device thereof 13 8,081,098 Integrator, delta-sigma modulator, analog-to-digital converter and applications thereof 14 8.035,439 Multi-channel integrator 15 8,031,404 TFly's eye integrator, illuminator, lithographic apparatus and method 16 8.029,144 Color mixing rod integrator in a laser-based projector 17 8.028,304 Component integrator 18 8,013,657 Temperature compensated integrator 19 8.011.810 Light integrator for more than one lamp 20 7,997,740 Integrator unit 21 7.965.795 Prevention of integrator wind-up in PI type controllers

24 7,943,893 Illumination optical system and image projection device having a rod integrator uniformizing spatial energy distribution of diffused illumination beam

23 7,954,962 Laser image display, and optical integrator and laser light source package used in such laser image display

Apr 26 2011

22 7,965,151 Pulse width modulator with two-way integrator

25 7,933,812 System integrator and commodity roll-up

26 7,932,960 Integrator array for HUD backlighting 27 7.911.256 Dual integrator circuit for analog front end (AFE) 28 7,907,115 Digitally synchronized integrator for noise rejection in system using PWM dimming signals to control brightness of cold cathode fluorescent lamp for backlighting liquid crystal display 29 7.905.631 Illumination system having coherent light source and integrator rotatable transverse the illumination axis 30 7.884.662 Multi-channel integrator 31 7,880,969 Optical integrator for an illumination system of a microlithographic projection exposure apparatus 32 7,873,223 Cognition integrator and language 33 7,834,963 Optical integrator 34 7,830,197 Adjustable integrator using a single capacitance 35 RE41,792 Controllable integrator 36 7.788,309 Interleaved comb and integrator filter structures 37 7.773,730 Voice record integrator 38 7,729,577 Waveguide-optical Kohler integrator utilizing geodesic lenses 39 7,726,819 T Structure for protecting a rod integrator having a light shield plate with an opening 40 7.724.063 Integrator-based common-mode stabilization technique for pseudo-differential switched-capacitor circuits 41 7.714.634 ■ Pseudo-differential active RC integrator 42 7.706.072 Optical integrator, illumination optical device, photolithograph, photolithography, and method for fabricating device 43 7,696,913 Signal processing system using delta-sigma modulation having an internal stabilizer path with direct output-to-integrator connection 44 7.693,430 Burst optical receiver with AC coupling and integrator feedback network 45 7.679.540 Double sampling DAC and integrator 46 7,671,774 Analog-to-digital converter with integrator circuit for overload recovery 47 7.658.497 Rod integrator holder and projection type video display 48 7,629,917 Integrator and cyclic AD converter using the same 49 7.619.550 Delta-sigma AD converter apparatus using delta-sigma modulator circuit provided with reset circuit resetting integrator 50 7.611.246 Projection display and optical integrator

#### Nov 3 2009

- PAT. NO. Title
- 51 7.605.645 Transconductor, integrator, and filter circuit
- 52 7,599,631 ■Burst optical receiver with AC coupling and integrator feedback network
- 53 7,575,159 Point of sale integrator
- 54 7.570.032 Regulator with integrator in feedback signal
- 55 7,565,326 Dialect independent multi-dimensional integrator using a normalized language platform and secure controlled access
- 56 7.554,400 Integrator and error amplifier
- 57 7.543.945 Integrator module with a collimator and a compact light source and projection display having the same
- 58 7.532,145 High resolution and wide dynamic range integrator
- 59 7.528.818 Digitally synchronized integrator for noise rejection in system using PWM dimming signals to control brightness of light source
- 60 7.511.648 Integrating/SAR ADC and method with low integrator swing and low complexity
- 61 7,474,241 Delta-sigma modulator provided with a charge sharing integrator
- 62 7.471.456 Optical integrator, illumination optical device, exposure device, and exposure method
- 63 7.454.750 Integrator adaptor and proxy based composite application provisioning method and apparatus
- 64 7.447.049 Single ended flyback power supply controllers with integrator to integrate the difference between feedback signal a reference signal
- 65 7,423,729 

  Method of monitoring the light integrator of a photolithography system
- 66 7.417.485 ■Differential energy difference integrator
- 67 7,415,716 Component integrator
- 68 7.411.534 Analog-to-digital converter (ADC) having integrator dither injection and quantizer output compensation
- 69 7,411,198 ■Integrator circuitry for single channel radiation detector
- 70 7,395,090 Personal portable integrator for music player and mobile phone
- 71 7.385.426 Low current offset integrator with signal independent low input capacitance buffer circuit
- 72 7.379.160 Optical integrator, illumination optical device, exposure apparatus, and exposure method
- 73 7.352.510 Light-pipe integrator for uniform irradiance and intensity
- 74 7,345,285 Spectra acquisition system with threshold adaptation integrator
- 75 7.333.626 Arbitrary coverage angle sound integrator

- 76 7,324,654 Arbitrary coverage angle sound integrator
- 77 7.324,025 Non-integer interpolation using cascaded integrator-comb filter
- 78 7,315,268 Integrator current matching
- 79 7,304,592 Method of adding a dither signal in output to the last integrator of a sigma-delta converter and relative sigma-delta converter
- 80 7,280,405 Integrator-based current sensing circuit for reading memory cells
- 81 7,262,056 Enhancing intermolecular integration of nucleic acids using integrator complexes
- 82 7.243,844 Point of sale integrator
- 83 7,242,333 Alternate sampling integrator
- 84 7,205,849 Phase locked loop including an integrator-free loop filter
- 85 7,187,948 Personal portable integrator for music player and mobile phone
- 86 7,182,468 Dual lamp illumination system using multiple integrator rods
- 87 7,180,357 Operational amplifier integrator
- 88 7,170,959 Tailored response cascaded integrator comb digital filter and methodology for parallel integrator processing
- 89 7,155,470 Variable gain integrator
- 90 7.152,981 Projection illumination system with tunnel integrator and field lens
- 91 7,152,084 Parallelized infinite impulse response (IIR) and integrator filters
- 92 7,150,968 Bridging INtegrator-2 (Bin2) nucleic acid molecules and proteins and uses therefor
- 93 7,138,848 Switched capacitor integrator system
- 94 7,130,764 Robust DSP integrator for accelerometer signals
- 95 7,102,844 Dual direction integrator for constant velocity control for an actuator using sampled back EMF control
- 96 7,102,548 Cascaded integrator comb filter with arbitrary integer decimation value and scaling for unity gain
- 97 7.098,845 Apparatus for generating an integrator timing reference from a local oscillator signal
- 98 7.098.827 Integrator circuit
- 99 7,098,718 Tunable current-mode integrator for low-frequency filters
- 100 7,087,881 Solid state image pickup device including an integrator with a variable reference potential

1	PAT. NO.	Title
		Wind turbine generator having integrator tracking
		Multiple rate discharge circuit for integrator, especially for use in computerized axial tomography
		Continuous integrator control linkage
		Differential integrator
		Circuit for light-integrator-controlled electronic flash unit
		Integrator circuits for a constant velocity vector generator
		Dual integrator EEG analyzer
		Automatic control system with integrator offset
		Temperature function integrator
		Logic controlled integrator
		System and method for operating a steam turbine with digital computer control having integrator limit
		Multiple-time-constant integrator or differentiator
		Electronic integrator for chart recorder
		Multiple dumping integrator
		Automatic scaled digital integrator
		Doppler detection device with integrator sampling means to inhibit false alarms
		Fast reset integrator
		Electrostatic transducer and acoustic and electric signal integrator
		Low friction absolute pressure continuous integrator
		Field effect transistor Miller integrator oscillator with temperature compensating impedance
		Noise immune reset circuit for resetting the integrator of an electronic engine spark timing controller
		Digital signal processing arrangement using a cascaded integrator function generator
		Bidirectional reset integrator converter
		Electronic integrator for gas volume calculations
		Watt/watthour transducer and integrator and current sources therefor
		High capacity recirculating delay loop integrator
		Heat unit integrator for X-ray tubes
		Barometrically compensated pressure index continuous integrator for measuring throughput fluid flow of meters
		Signal generator for electronic musical instrument, employing variable rate integrator
		Low frequency two phase oscillator including variable feedback integrator circuits
Jan 1976 531	3,931,619	Overtemperature monitor and integrator apparatus



Stay Safe and Stay Healthy!

## End of Lecture 23